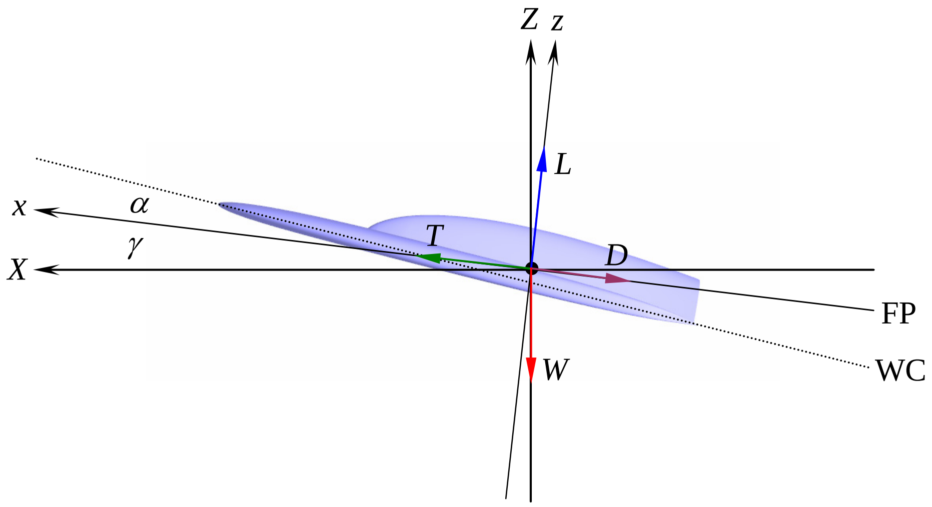
# Space Plane Ascent Path Computation




$L$ = Lift $\qquad$ $T$ = Thrust $\qquad$ $\alpha$ = Attack Angle $\qquad$ FP = Flight Path

$D$ = Drag $\qquad$ $W$ = Weight $\qquad$ $\gamma$ = Climb Angle $\qquad$ WC = Wing Chord

Summation of forces in the mass centered local horizon frame $(X, Z)$ yields,

$$T\cos\gamma - D\cos\gamma - L\sin\gamma = m\,a\cos\gamma$$

$$T\sin\gamma - D\sin\gamma + L\cos\gamma - W = m\,a\sin\gamma - m\frac{(v\cdot\cos\gamma)^2}{(R_e + h)}$$

Since the local horizon frame assumes a circular path, there is an apparent inertial lift $I$, which is a function of flight speed $v$ and altitude $h$, while $R_e$ is Earth's radius.

$$I = m\frac{(v\cdot\cos\gamma)^2}{(R_e + h)} \qquad \text{also} \qquad W = m\,g_h$$

where $\qquad g_h = g_e\left(\frac{R_e}{R_e + h}\right)^2 \qquad g_e = 9.807\,\text{m/s}^2 \qquad R_e = 6378\,\text{km}$

Using $a_X = a\cos\gamma$ and $a_Z = a\sin\gamma$, instantaneous mass $m(t)$, and solving for acceleration,

$$\vec{a} = \begin{pmatrix} a_X \\ a_Z \end{pmatrix} \qquad \vec{a} = a\begin{pmatrix} \cos\gamma \\ \sin\gamma \end{pmatrix}$$

$$a_X = \frac{T\cos\gamma - D\cos\gamma - L\sin\gamma}{m(t)}$$

$$a_Z = \frac{T\sin\gamma - D\sin\gamma + L\cos\gamma}{m(t)} + \frac{(v\cdot\cos\gamma)^2}{(R_e + h)} - g_h$$

The instantaneous velocity vector is always tangential to the flight path,

$$\vec{v} = \begin{pmatrix} v_X \\ v_Z \end{pmatrix} \qquad \vec{v} = v\begin{pmatrix} \cos\gamma \\ \sin\gamma \end{pmatrix}$$

The instantaneous position vector assumes a flat Earth and points from the runway takeoff location projected onto sea level to the vehicle's center of mass. At time $t = 0$, the vehicle is on the runway and is about to move, thus distance traveled $s = 0,\ h = h_0$ and $v = 0$.

$$\vec{r} = \begin{pmatrix} r_X \\ r_Z \end{pmatrix} \qquad \vec{r}(t) = \begin{pmatrix} s \\ h \end{pmatrix}$$

The position and velocity vectors are integrated in time using a small time step $\Delta t$ until $h = 120$ km is reached. The acceleration vector is updated accordingly at each new time step.

$$\vec{r}(t+\Delta t) = \vec{r}(t) + \vec{v}(t)\cdot\Delta t$$

$$\vec{v}(t+\Delta t) = \vec{v}(t) + \vec{a}(t)\cdot\Delta t$$

$$\vec{a}(t+\Delta t) = f\left[\vec{r}(t+\Delta t), \vec{v}(t+\Delta t), m(t+\Delta t)\right]$$

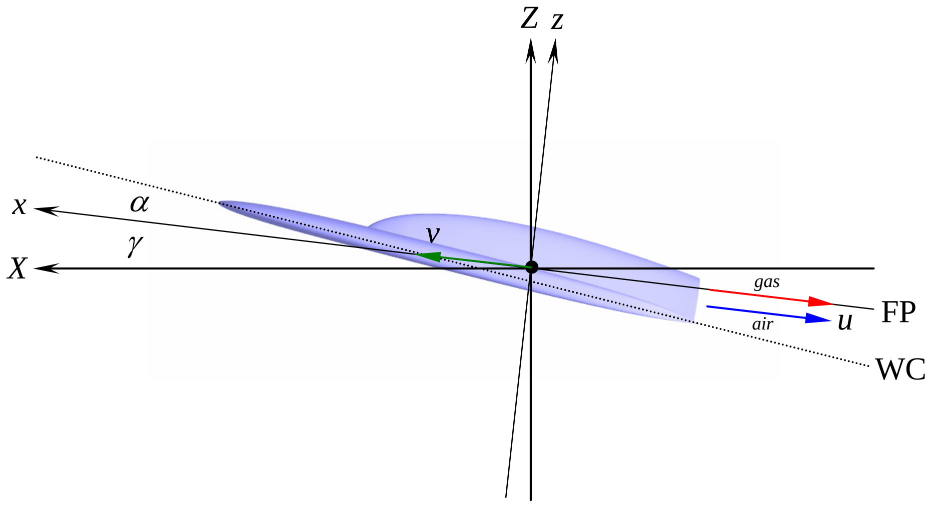
$$m(t+\Delta t) = m(t) - \dot{m}_{fuel}\cdot\Delta t \qquad \dot{m}_{fuel} = \frac{T}{I_{sp}\cdot g_e} \qquad \dot{m}_{fuel} = \dot{m}_{\mathrm{LH_2}} + \dot{m}_{\mathrm{LO_2}} = \dot{m}_{gas}$$

Lift and drag are calculated from their respective coefficients, which are both a function of attack angle and Mach number, as well as the dynamic pressure $p_{dyn}$ and wing area $A_w$.

$$L = C_L(M,\alpha)\ p_{dyn}\ A_w \qquad D = C_D(M,\alpha)\ p_{dyn}\ A_w$$

where $\quad p_{dyn} = \frac{1}{2} k\, M^2 p \qquad M = \dfrac{v}{\sqrt{k\,R\,\theta}}$

The Mach number term in the denominator is the speed of sound, which is a function of local temperature $\theta$, gas constant $R$, and ratio of specific heats $k$. Local pressure and temperature $(p,\theta)$ are defined by the 1976 US Standard Atmosphere (USSA) as a function of altitude $h$. Above 86 km altitude air changes its composition, and thus $R$ and $k$ change gradually.




$\alpha$ = Attack Angle
$\gamma$ = Climb Angle
FP = Flight Path
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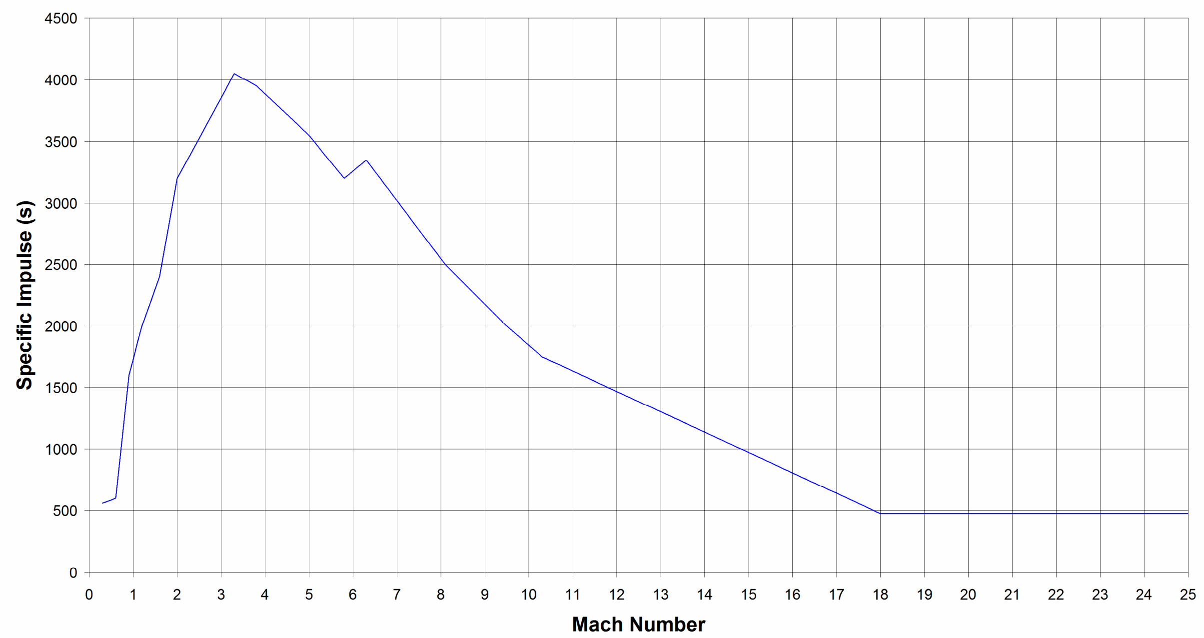
The exhaust and its resulting thrust are directed along the flight path. The hot gas, generated by the combustion of onboard liquid hydrogen and oxygen, can be mixed with incoming cold air to achieve a higher specific impulse $I_{sp}$ in a rocket based combined cycle (RBCC) engine. An $I_{sp}$ profile as a function of Mach number for the proposed RBCC engine is shown below.

Specific Impulse versus Flight Mach Number




The thrust $T$ of the RBCC engine is dictated primarily by the size of its intake and exhaust areas, and the variable propellant flow feeding it. Thrust shall be limited at all times to exert no more than 1.8 $g_e$ of total acceleration, gravity included, on the vehicle and its occupants.